4.3 What are the derivatives of exponential and logarithmic functions?

2014-11-13 day 54

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$$\frac{d}{dx}(e^x) = e^x$$

$$\frac{d}{dx}(b^x) = b^x \cdot (\ln b)$$

[[but remember....]]

$$b^x = (e^{\ln b})^x = e^{x \ln b}$$

$$\frac{d}{dx}(e^{x\ln b}) = e^{x \ln b} \cdot \frac{d}{dx}(x \ln b)$$

$$= e^{x \ln b} \cdot \ln b = b^x \cdot \ln b$$

$$\boxed{\frac{d}{dx}(\ln u) = \frac{1}{u} \cdot \frac{du}{dx}}$$

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$ [[but..]]

$$\frac{d}{dx}(\log_b x) = \frac{1}{x \ln b}$$

[[duh]].

$$b^{\log_b x} = x$$

$$\frac{d}{dx}(c\, f(x)) = c \frac{d}{dx}(f(x))$$

$$\frac{d}{dx}(x \ln b) = \underbrace{\frac{d}{dx}(x)}_{1} \cdot \ln b + x \underbrace{\frac{d}{dx}(\ln b)}_{0}$$

$$= \ln b$$

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"the problem" $a^b = c \Leftrightarrow \log_a c = b$

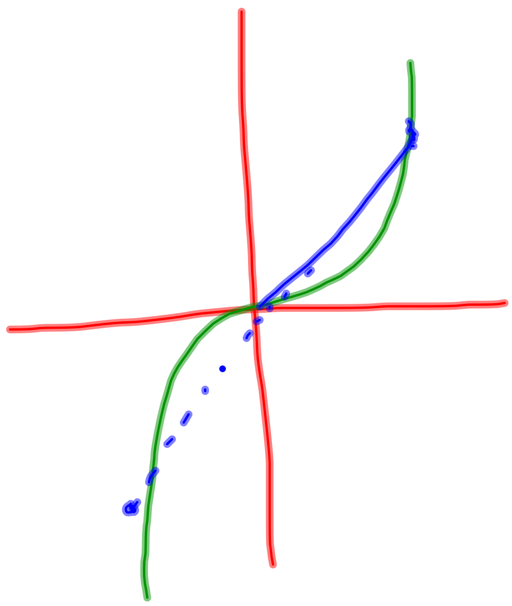
the domain of $\ln x$ and all the related logarithms is $x > 0$

it often helps to consider $\ln|x|$

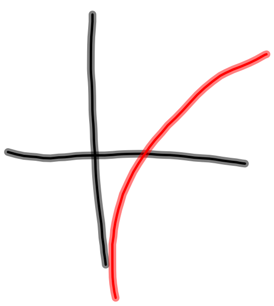
odd
$f(-x) = -f(x)$ → "symmetric about the origin"

180° rotational symmetry

$\sin x$
x^3
x^5
⋮



$\ln x$



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key mathematical ways of thinking

→ how do I undo this . . . . .

[going backwards]

→ how can I generalize this?

[can I get a more specific example to try & understand]

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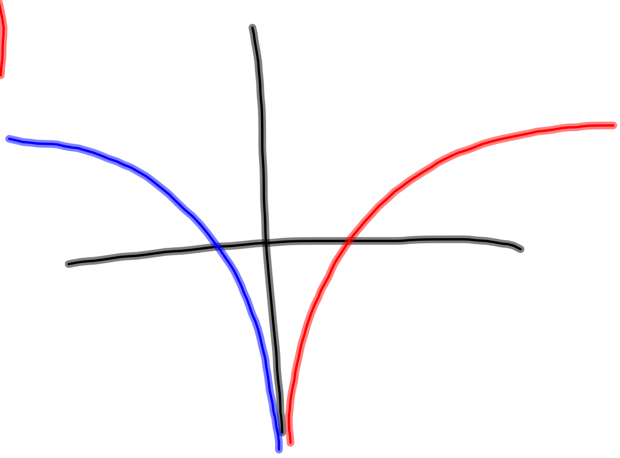
$\ln|x|$



domain of $\ln|x|$ is ALL real numbers* [except x=0]

$$\ln|x| = \begin{cases} \ln x, & x>0 \\ \ln(-x), & x<0 \end{cases}$$

$x \to -x \to \ln(\ )$

$$\frac{d}{dx}(\ln|x|) = \begin{cases} \frac{1}{x}, & x>0 \\ \frac{1}{(-x)} \cdot \frac{d}{dx}(-x), & x<0 \end{cases}$$

$$= \frac{1}{-x} \cdot (-1) = \frac{1}{x}, \ x<0$$

$$\frac{d}{dx}(\ln|x|) = \frac{1}{x}, \ x \neq 0$$

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$$\frac{d}{dx}\left(x^x\right)?$$

$$y = x^x$$

"ln" of both sides

$$\ln y = \ln\left(x^x\right) = x\ln x$$

$\frac{d}{dx}$:

$$\frac{1}{y}\cdot\frac{d}{dx}(y) = \underbrace{\frac{d}{dx}(x)}_{1}\cdot\ln x + x\cdot\underbrace{\frac{d}{dx}(\ln x)}_{\frac{1}{x}}$$

$$\frac{1}{y}\cdot\frac{dy}{dx} = 1\cdot\ln x + x\left(\frac{1}{x}\right)$$

$$y\left(\frac{1}{y}\frac{dy}{dx}\right) = \left(\ln x + 1\right)y$$

$$\frac{dy}{dx} = y\left(\ln x + 1\right) = x^x\left(\ln x + 1\right)$$

problem

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

$$\frac{d}{dx}\left(n^x\right) = n^x\cdot\ln(n)$$

how do we reconcile these?

$$\left(-\frac{1}{2}\right)^{-\frac{1}{2}} = \frac{1}{\left(-\frac{1}{2}\right)^{1/2}} = \frac{1}{\sqrt{-\frac{1}{2}}}$$

undefined

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0^0 "indeterminate"

$\lim_{x \to 0} 0^x = 0$

$\lim_{x \to 0} x^0 = 1$

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$$\$ = \boxed{1}^{P}\left(1+\frac{1}{t}\right)^{t}$$

$$A = P\left(1+\frac{r}{p}\right)^{p \cdot y}$$

compound	amount
1	\$2
2	\$2.25
10	$(1.1)^{10} =$ \$2.59
100	$(1.01)^{100} =$ \$2.7048...
1000	$(1.001)^{1000} =$ \$2.7169
1000000	$(1.000001)^{1000000} = 2.7182$

Compound continuously: Pe^{rt}

$e = 2.718281828459\ldots$

$$e = \lim_{x \to \infty}\left(1+\frac{1}{x}\right)^{x} = \lim_{x \to 0}(1+x)^{\frac{1}{x}}$$

$\pi = \{0,1,2,3,4,5,6,7,8,9$ rearrange & duplicate as necessary$\}$

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Fibonacci seq.

0, 1, 1, 2, 3, 5, 8, 13.

$1+1=2$ $\quad 3+5=8$

$1+2=3$ $\quad 5+8=13$

$2+3=5$

Lucas sequence

$L_0 = .1$ $\quad -\pi$

$L_1 = 5$ $\quad \pi$

$L_n = L_{n-1} + L_{n-2}$

$L_2 = L_1 + L_0$

$L_2 = 5 + .1 = 5.1$